AUGUST 2023
EBS 301
CALCULUS
2 HOURS

Candidate's Index Number
Signature:

UNIVERSITY OF CAPE COAST
COLLEGE OF EDUCATION STUDIES
SCHOOL OF EDUCATIONAL DEVELOPMENT AND OUTREACH
INSTITUTE OF EDUCATION

COLLEGES OF EDUCATION
FOUR-YEAR BACHELOR OF EDUCATION (B.ED)
THIRD YEAR, END-OF-FIRST SEMESTER EXAMINATION, AUGUST 2023

16TH AUGUST 2023 **CALCULUS** **9:00 AM – 9:40 AM**

This paper consists of two sections, A and B. Answer ALL the questions in Section A and TWO questions from Section B. Section A will be collected after the first 40 minutes.

SECTION A
(20 MARKS)

Answer ALL questions in this Section.

Items 1 to 20 are stems followed by four options lettered A to D. Read each item carefully and circle the letter of the correct or best option.

1. Given $\lim_{x\to a} f(x) = A$, what is $\lim_{x\to a} cf(x)$?
   A. A
   B. A − c
   C. A + c
   D. cA

2. Evaluate $\lim_{x\to -1} \frac{\ln x}{x+1}$.
   A. -1
   B. 0
   C. 1
   D. Does not exist

3. Evaluate $\lim_{x\to 4} \sqrt{25 - x^2}$.
   A. 0
   B. 3
   C. 6
   D. 9

4. Determine the point(s) where the function $f(x) = \frac{x^2-4}{x-2}$ is **not** continuous.

A. 2
B. 3
C. 4
D. 5

5. Which of the following statements is **not** true for a function $f(x)$ to be continuous at x_0?

A. $f(x_0)$ is defined
B. $\lim_{x \to x_0} f(x)$ exists
C. $\lim_{x \to x_0} f(x) = f(x_0)$
D. $\lim_{x \to x_0} f(x) \neq f(x_0)$

6. Given that u and v are functions of x, find $\frac{d(uv)}{dx}$.

A. $\frac{du}{dx} + \frac{dv}{dx}$
B. $\frac{dudv}{dx^2}$
C. $u\frac{dv}{dx} + v\frac{du}{dx}$
D. $\frac{du}{dx} \cdot \frac{dv}{dx}$

7. If $y = f(x)$ and $u = g(x)$ are both differentiable functions, then $\frac{dy}{dx}$ is given as .....

A. $\frac{dy}{du} + \frac{dx}{du}$
B. $\frac{dy}{du} \cdot \frac{du}{dx}$
C. $\frac{dydx}{du^2}$
D. $\frac{dy}{du} - \frac{dx}{du}$

8. Find with respect to x, the first derivative of $(2 - 3x)^5$.

A. $-15(2 - 3x)^5$
B. $15(2 + 3x)^4$
C. $-15(2 - 3x)^4$
D. $-15x(2 - 3x)^4$

9. Find the linearization $L(x)$ of the function $f(x) = \sqrt{x}$ at $a = 4$.

A. $1 + \frac{1}{4}x$
B. $1 + \frac{1}{2}x$
C. $2 + \frac{1}{4}x$
D. $2 + \frac{1}{2}x$

10. If the linearization of $g(x) = \sqrt[3]{1+x}$ at $a = 0$ is $L(x) = 1 + \frac{1}{2}x$, use it to estimate the value of $\sqrt[3]{1.1}$.
    A. 1.02
    B. 1.03
    C. 1.04
    D. 1.05

11. Which of the following statement is **not** a condition for the function f to be continuous at $x = a$?
    A. $f(a)$ is defined.
    B. $\lim_{x \to a} f(x)$ exists.
    C. $\lim_{x \to a} f(x) = f(a)$
    D. $\lim_{x \to a} f(a) = f(x)$

12. Find $\lim_{x \to 1} \frac{\ln x}{x-1}$.
    A. 0
    B. 1
    C. 2
    D. ∞

13. Evaluate $\int_0^2 (4t + 3)dt$.
    A. 14
    B. 15
    C. 17
    D. 20

14. Let m_1 and m_2 be the gradients of the tangent and normal lines respectively to a curve at a point. What is $m_1 m_2$?
    A. -2
    B. -1
    C. 0
    D. 1

15. Find the equation of the normal line to $y = x^3 - 2x^2 + 4$ at $(2,\ 4)$.
    A. $4x - y = 18$
    B. $x - 4y = 18$
    C. $x + 4y = 18$
    D. $4x + y = 18$

16. The velocity of a particle in ms^{-1}, after t seconds, $v = 3t^2 + 2t - 1$. Determine its acceleration at the of 2 seconds.
    A. $10m/s^2$
    B. $13m/s^2$
    C. $14m/s^2$
    D. $17m/s^2$

17. If $\int_2^a (2x + 2)\, dx = 8$, $a > 0$, determine the value of a.

A. 2
B. 3
C. 6
D. 8

18. Find the area under the parabola $y = x^2 + 2$, above the $x-$ axis, and between $x = 1$ and $x = 3$.

A. $10\frac{2}{3}$ sq units
B. $12\frac{2}{3}$ sq units
C. $12\frac{2}{7}$ sq units
D. $12\frac{5}{7}$ sq units

19. Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$.

A. $\frac{1}{3}$
B. $\frac{1}{2}$
C. 2
D. 3

20. Find y'', given $x + xy + y = 2$.

A. $\frac{1+y}{(1+x)^2}$
B. $\frac{2(1+y)}{(1+x)^2}$
C. $\frac{1-y}{(1+x)^2}$
D. $\frac{2(1-y)}{(1+x)^2}$